647510 **BSc5MatC501x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination

Oct/Nov - 2022 (NEW COURSE)

Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any two of the following questions. [10]

1. Let $(X,\ d)$ be a metric space. Define another metric d' on X using metric d and prove your claim.
2. Prove that arbitrary union of open sets in a metric space X is open in X. Does the result hold for closed sets? Justify.
3. Define Cantor set $C$and check whether $\frac{4}{5} \in C$ or not.

Q.1 (B) Answer any one of the following questions. [04]

1. Check whether 0 is a limit point of the set $E = (0,\ 1)$ or not in the discrete metric space $(R,\ d)$.
2. Prove that in any metric space a neighborhood of a point is an open set.

Q.2 (A) Answer any two of the following questions. [10]

1. State and prove necessary and sufficient condition for a bounded function *f* on *[a, b]* to be R-integrable.
2. Let $f(x) = x^2, x \in [-1, 1]$ and $P = \left\{-1, -\frac{1}{2}, 0, \frac{3}{4}, 1\right\}$. Find $L(P, f)$ and $U(P, f)$.
3. Prove that | f | is R-integrable if *f* is R-integrable. Is the converse true? Justify.

Q.2 (B) Answer any one of the following questions. [04]

1. Show that $f(x) = \cos x$ is R-integrable on $\left[0, \frac{\pi}{2}\right]$ and evaluate $\int_0^{\frac{\pi}{2}} \cos x\ dx$.
2. Let $f : [a, b] \to R$ be a bounded function. In usual notations show that $\underline{\int_a^b} f(x)dx \le \overline{\int_a^b} f(x)\ dx$.

Q.3 (A) Answer any two of the following questions. [10]

1. Verify first mean value theorem for $f(x) = x^2$ on [-1, 1].
2. Let *f* be an integrable function on *[a, b]*. Define integral function of *f* and show that it is continuous on *[a, b]*.
3. Evaluate $\lim_{n\to\infty} \left[\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)\left(1 + \frac{3}{n}\right) . . . \left(1 + \frac{4n}{n}\right)\right]^{\frac{1}{n}}$.

Q.3 (B) Answer any one of the following questions. [04]

1. For $0 < a < b$, prove that $\frac{\pi^3}{3a} \le \int_0^\pi \frac{x^2}{a\cos^2\frac{x}{2} + b\sin^2\frac{x}{2}} dx \le \frac{\pi^3}{3b}$.
2. Determine whether the binary operation $*$ defined as $a * b = a^b; \forall a, b \in Z^+$ is associative or not.

Q.4 (A) Answer any two of the following questions. [10]

1. Let $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in R\backslash\{0\} \right\}$. Show that G is a group with usual matrix multiplication.
2. Prove that a Group cannot be a union of its two proper subgroups.
3. State Euler's theorem and using it find reminder when $3^{257}$is divided by 14.

Q.4 (B) Answer any one of the following questions. [04]

1. Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 8 & 5 & 4 & 6 & 9 & 3 & 10 & 7 & 2 & 1 \end{pmatrix}$. Check whether the permutation *f* is even or odd and also find its order.
2. Let G be a group and $a \in G$. Then in usual notations prove that $O(a^m) \leq O(a), \forall m \in Z$ and $O(bab^{-1}) = O(a); \forall\, b \in G$.

Q.5 (A) Answer any two of the following questions. [10]

1. Let $\phi: (G,*) \rightarrow (G',*')$ be an isomorphism. Show that $\phi(N)$ is a normal subgroup of G' if N is a normal subgroup of G.
2. Show that isomorphism of groups is a transitive relation.
3. State and prove Cayley's theorem.

Q.5 (B) Answer any one of the following questions. [04]

1. Prove that a subgroup of index 2 of a group is a normal subgroup.
2. Let H be a normal subgroup of a finite group G. Prove that *o(Ha)/o(a)*, $\forall a \in G$.

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